



# Novel Whispering-Gallery Resonators for Oscillators, Modulation and Sensing

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*Contributions and collaborations acknowledged*

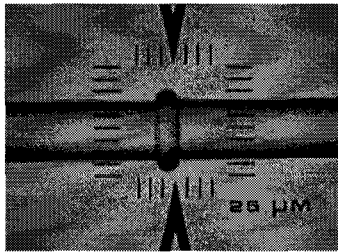
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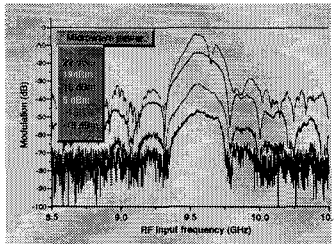
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### 1. Microtorus: a novel high-finesse microcavity

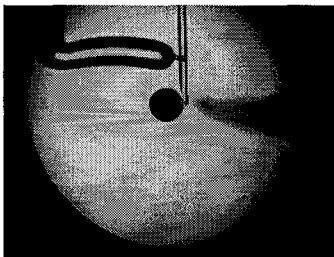
The highly oblate spheroidal dielectric microcavity combines very high Q-factor, typical of microspheres, with vastly reduced number of excited whispering-gallery (WG) modes



### 2. Microwave modulation using whispering-gallery modes

Spherical cavity of lithium niobate serves as a core for low-controlling-power electrooptical modulator

### 3. WG microcavity as a sensor: detection of nanomole dopants in fluids



Very high intrinsic Q of immersed microspheres allows for detection of dilute absorptive species in the evanescent field



## Microspheres have been here

low material loss (transparent material)

low bending loss (high-contrast boundary)

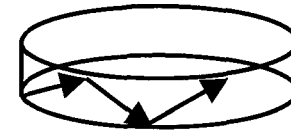
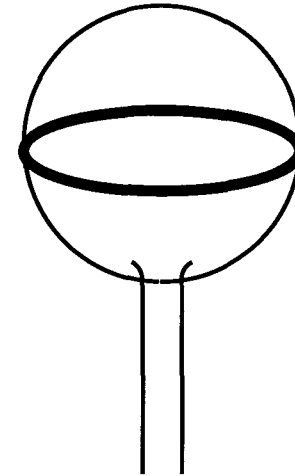
**LOW SCATTERING LOSS** (TIR always under grazing incidence)  $\Theta \rightarrow \pi/2$ ; compare to disks/ rings:

$$\frac{I_R}{I_I} = e^{-\left(\frac{4\pi\sigma}{\lambda} \cos \Theta\right)^2} \quad (\text{J.W.S.Rayleigh})$$

EVEN WITH MOLECULAR ROUGHNESS  $\sigma$ , ONLY CURVATURE CONFINEMENT ALLOWS **Q** LIMITED BY MATERIAL ATTENUATION:

$10^8 - 10^{10}$  in spheres vs.  $10^3 - 10^5$  in microrings !!

Drawback: “too many modes” compared to planar rings!

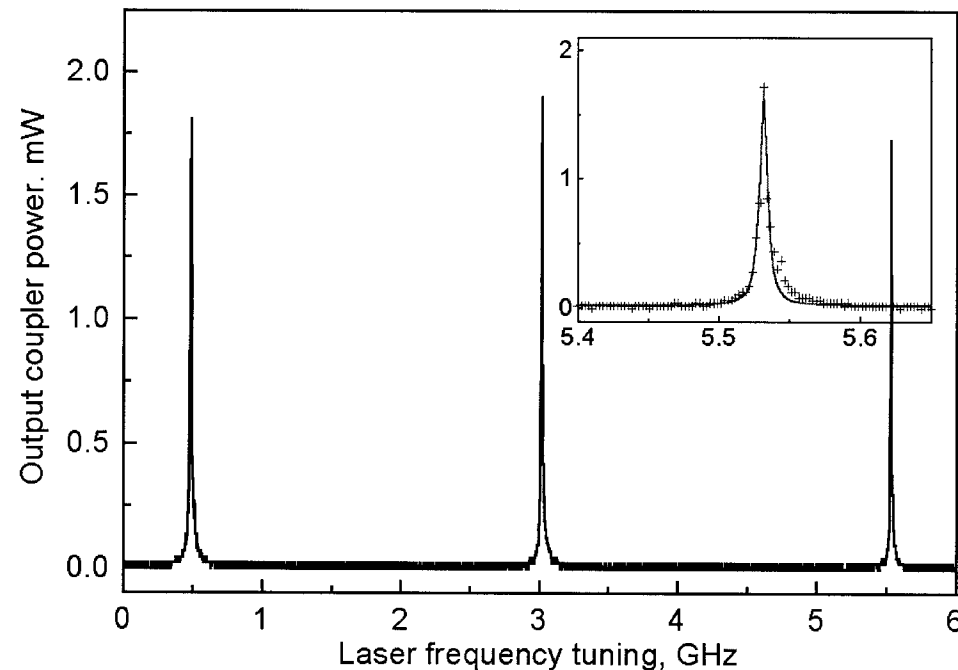
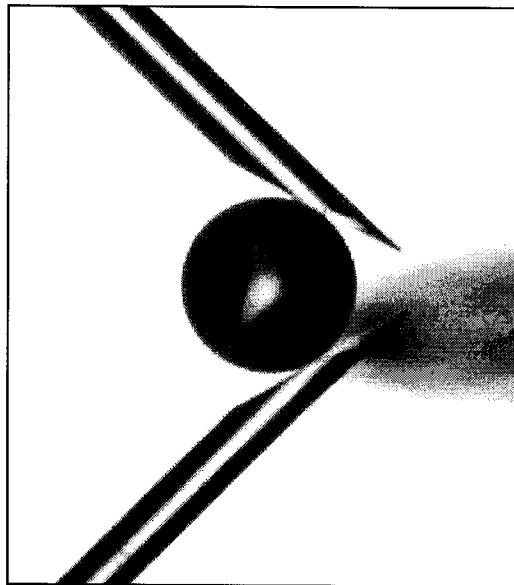




## Spectrum of microspheres: Families of non-degenerate $TE(TM)_{lmq}$ modes.

“Small” FSR  $\nu_{lmq} - \nu_{l,m-1,q} \sim \nu \frac{\varepsilon^2}{2l}$  - few GHz with typical  $\varepsilon^2 \sim (1-3) \times 10^{-2}$ .

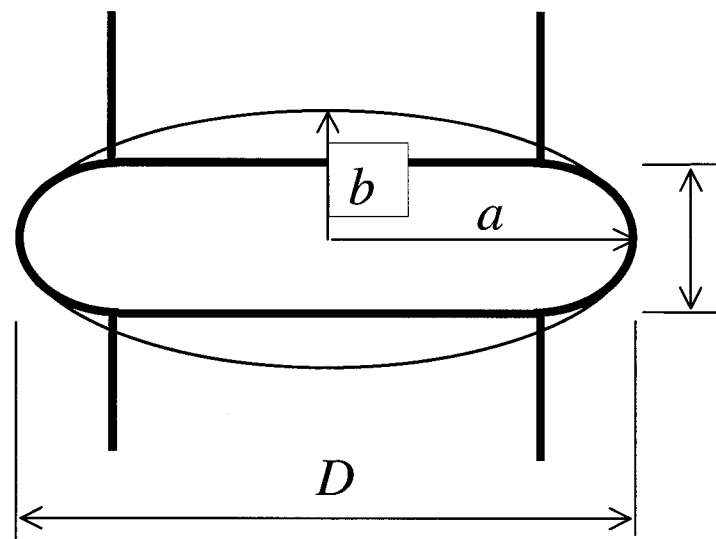
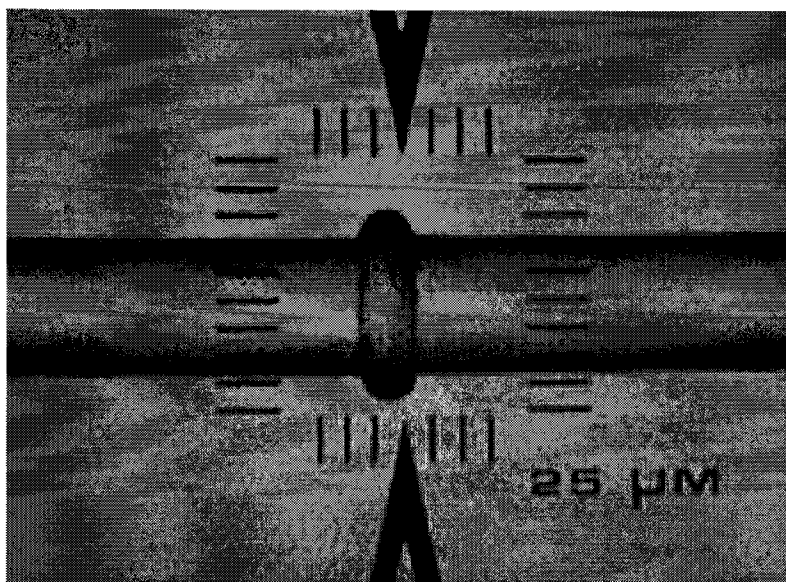
“Big” FSR  $\nu_{lmq} - \nu_{l-1,mq} = \frac{c}{2\pi na} (t_{lq} - t_{l-1,q}) \sim \nu / l$  - few hundred GHz (few nm)



Input power 7.5...8.3mW; maximum transmission at resonance  $\sim 23.5\%$  (fiber-to-fiber loss 6.3dB);  $Q_{load} > 3 \times 10^7$  at 1550nm; sphere diameter 405 $\mu$ m. Unloaded  $Q_o \approx 1.2 \times 10^8$  (*Opt.Lett.* 24, 723 (1999))



### Novel geometry: a highly oblate spheroid, or microtorus



Near the symmetry plane (at the location of WG modes), toroidal surface of outer diameter  $D$  and cross-section diameter  $d$  coincides with that of the osculating oblate spheroid with large semiaxis  $a = D / 2$  and small semiaxis  $b = \frac{1}{2} \sqrt{Dd}$



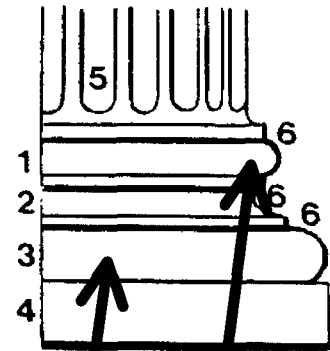
## Names

[F, seesaw]: an apparatus or structure  
end is counterbalanced by the other on  
or by weights

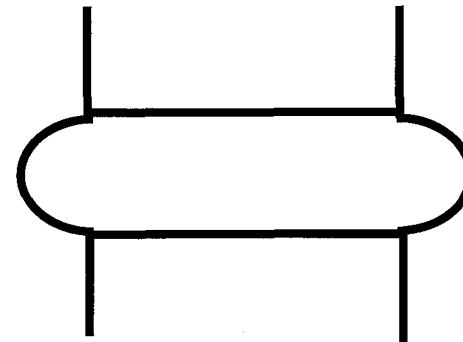
\'bā-səz\ [ME, fr.  
sp, base, fr. *bainein*

**a** : the bottom of  
support : FOUNDA-  
t of a wall, pier, or  
parate architectural  
part of a complete  
side or face of a  
which an altitude can  
on which the figure  
a bodily organ by  
other more central

**2 a** : a main in-  
latex ~> **b** : a  
ingredient (as of a  
ental part of some-  
the lower part of a  
oint or line from which a start is made  
b : a line in a survey which serves as



base of a column:  
1 upper torus, 2  
scotia, 3 lower  
torus, 4 plinth, 5  
shaft, 6 fillets



(Webster's New College dictionary; G & C Merriam Co. Springfield, Mass., 1975, p.92)



## Calculation of the spectrum of the dielectric spheroid

is not a trivial problem, even numerically. In “quasiclassical” approximation with assumptions:

1) a WG mode is a closed circular beam supported by TIR, 2) optical field tunnels outside at the depth  $1/k\sqrt{n^2-1}$ , and 3) the tangential component of  $E$  ( $TE$ -mode), or normal of  $D$  ( $TM$ -mode) is continuous at the boundary. Eigenfrequencies of high-order WG modes ( $l \gg 1; l \approx m$ ) in dielectric sphere can be approximated via solutions of scalar wave equation with zero boundary conditions, because most of the energy is concentrated in one component of the field ( $E_\theta$  for  $TE$ -mode and  $E_r$  for  $TM$ -mode).

Based on above considerations, let us estimate WG mode eigenfrequencies in oblate spheroids of large semiaxis  $a$ , small semiaxis  $b$ , and eccentricity  $\varepsilon = \sqrt{1-b^2/a^2}$ . Since WG modes are localized the “equatorial” plane, we shall approximate the radial distribution by cylindrical Bessel function  $J_m(n\tilde{k}_{mq}r)$  with  $n\tilde{k}_{mq}a = na\sqrt{k_{lmq}^2 - k_\perp^2} \approx T_{mq}$ , where  $J_m(T_{mq}) = 0$  and  $k_\perp$  is the wavenumber for quasiclassical solution for angular spheroidal functions. For our purposes a rough approximation is enough:  $k_\perp^2 \approx \frac{2(l-m)+1}{a^2\sqrt{1-\varepsilon^2}}m$ ;

more rigorous consideration can follow the approach given in [I.V.Komarov, L.I.Ponomarev, S.Yu.Slavyanov, *Spheroidal and Coulomb Spheroidal Functions*, Moscow, Nauka (1976) (in Russian)]. Taking into account that  $T_{mq} \approx t_{lq} - (l-m+1/2)$ , we finally obtain the following approximation:

$$nk_{lmq}a - \frac{\chi}{\sqrt{n^2-1}} \approx t_{lq} + \frac{2(l-m)+1}{2} \left( \frac{1}{\sqrt{1-\varepsilon^2}} - 1 \right)$$

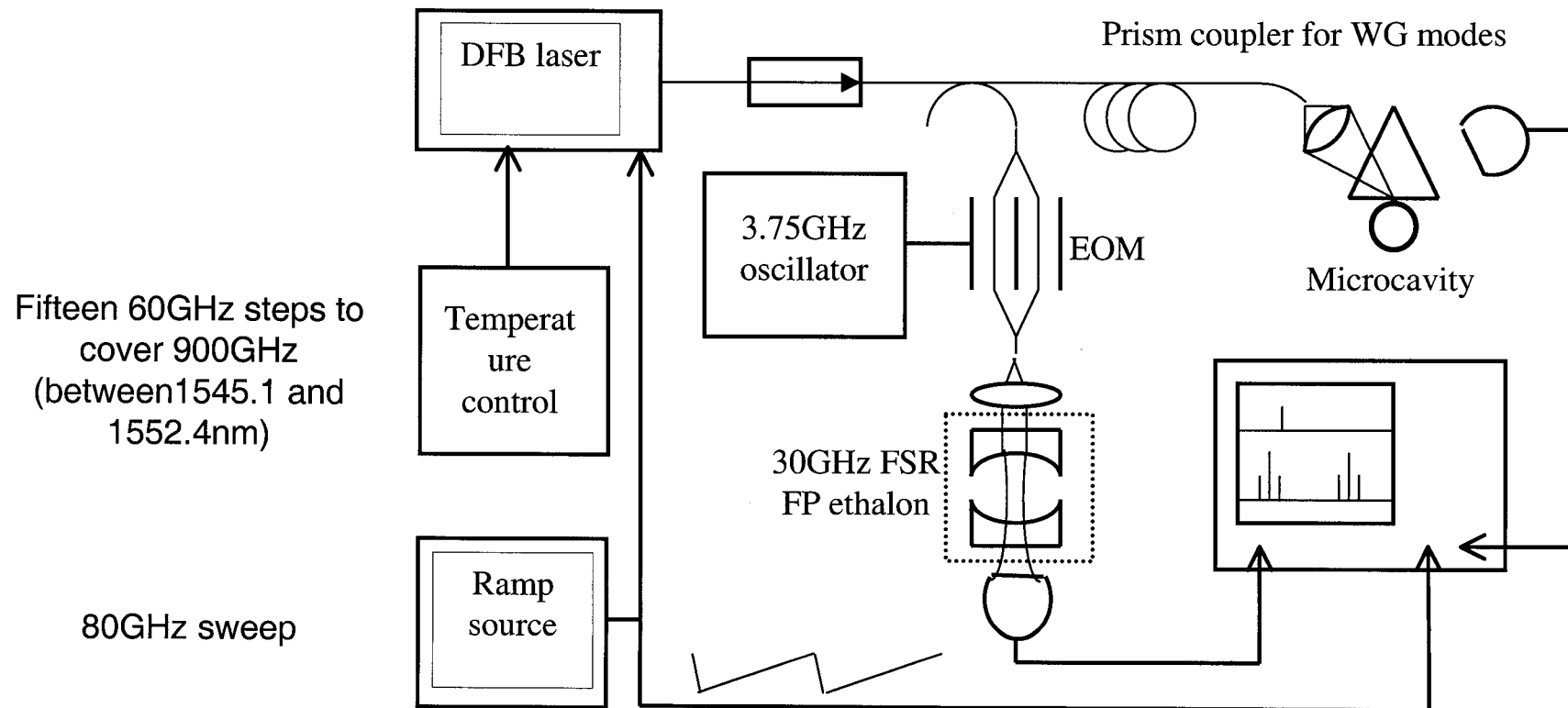
$t_{lq}$  --  $q$ -th zero of the spherical Bessel function of the order  $l$ ;  $\chi = n$  for  $TE$ -mode,  $\chi = 1/n$  for  $TM$ -mode.

1. For small eccentricities the model gives identical prediction with perturbation theory (H.M.Lai, P.T.Leung, K.Young, P.W.Barber, S.C.Hill, *Phys. Rev. A* 41, 5187-5198 (1990) )
2. Discrepancy with numerical calculations is <5% in prediction of “small” FSR and <0.1% of absolute frequencies, even with  $\varepsilon^2 \sim 0.8$ , even small  $l = 100$



## Schematic of the experimental setup

to obtain wide range ( $\sim 900\text{GHz}$ , or  $7.2\text{nm}$ ) high-resolution spectra of WG modes in microcavity



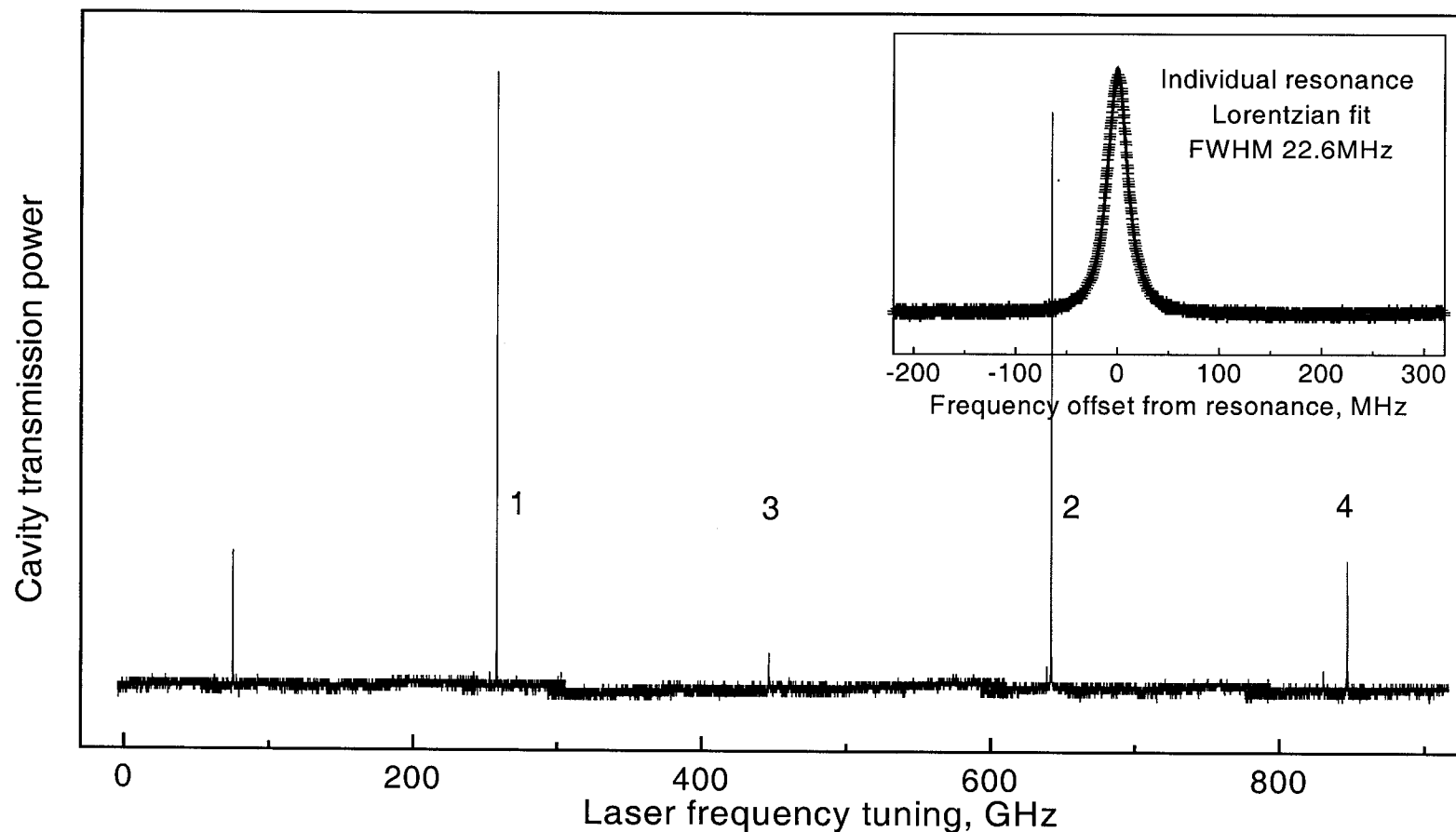




### Spectrum of $TE$ whispering-gallery modes in spheroidal dielectric microcavity

$D = 2a = 165\mu\text{m}$ ;  $d = 42\mu\text{m}$ ;  $2b = 83\mu\text{m}$ . Free spectral range (between largest peaks 1 and 2) 383.7GHz (3.06nm) near central wavelength 1550nm. Individual resonance bandwidth 23MHz (loaded  $Q = 8.5 \times 10^6$ ).

$$\text{Finesse } F = 1.7 \times 10^4$$





## Main experimental result: “small” FSR gets large

Our parameters:  $a = 82.5\mu\text{m}$ ,  $b = 42.5\mu\text{m}$  ( $\epsilon = 0.86$ ),  $n = 1.453$ ,  $\lambda \sim 1550\text{nm}$ ,  $TE$ -modes  $l \approx 473$

Estimate of “large” FSR (calculations)

$$\nu_{lmq} - \nu_{l-1,mq} = \frac{c}{2\pi na} (t_{lq} - t_{l-1,q}) = \frac{c}{2\pi na} (1 + 0.617l^{-2/3} + O(l^{-5/3})) \approx 402\text{GHz}$$

Estimate of “small” FSR

$$\nu_{lmq} - \nu_{l,m-1,q} = \frac{c}{2\pi na} \left( \frac{1}{\sqrt{1-\epsilon^2}} - 1 \right) \approx 382\text{GHz}$$

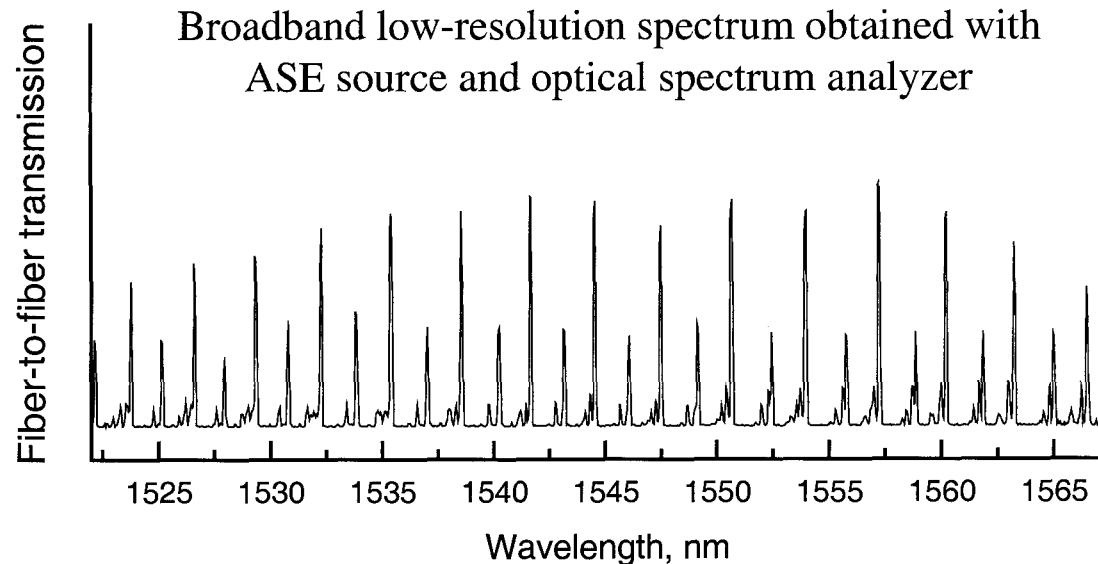
Experimental spectrum:	peaks 1-2	$383.7 \pm 0.5\text{ GHz}$
	peaks 3-4	$400.3 \pm 0.5\text{ GHz}$

Question yet to answer:

*exact mechanism of coupling roll-off for different  $l$  mode families*



Are there dispersion mechanisms to alter spectral character “in wide”?

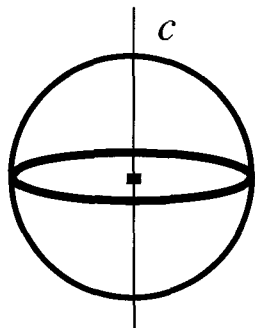


CONCLUSION

1. It seems indeed we can combine small size, ultra-high-Q with “nice” FP-like spectrum for true finesse  $10^4 - 10^6$  -- in microcavities as opposed to “super” mirror FPs
2. Complete “mode cleaning” can be expected with higher eccentricities;  
higher Qs -- with refinement of fabrication
3. Potential applications may be diverse

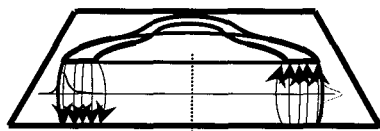


## Electro-optic modulation with whispering-gallery modes



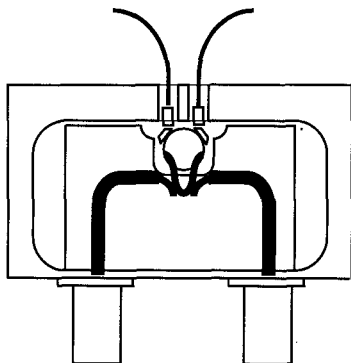
Optical whispering-gallery modes in lithium niobate sphere  
in the perpendicular plane to principal crystal axis.

Optical FSR is the operational microwave frequency



Equatorial layer is sliced out of the sphere  
and microwave cavity(ies) is (are) built around.

Variation of microwave field phase along the circumference is required  
to satisfy momentum conservation in 3 photon process by  $\chi^{(2)}$



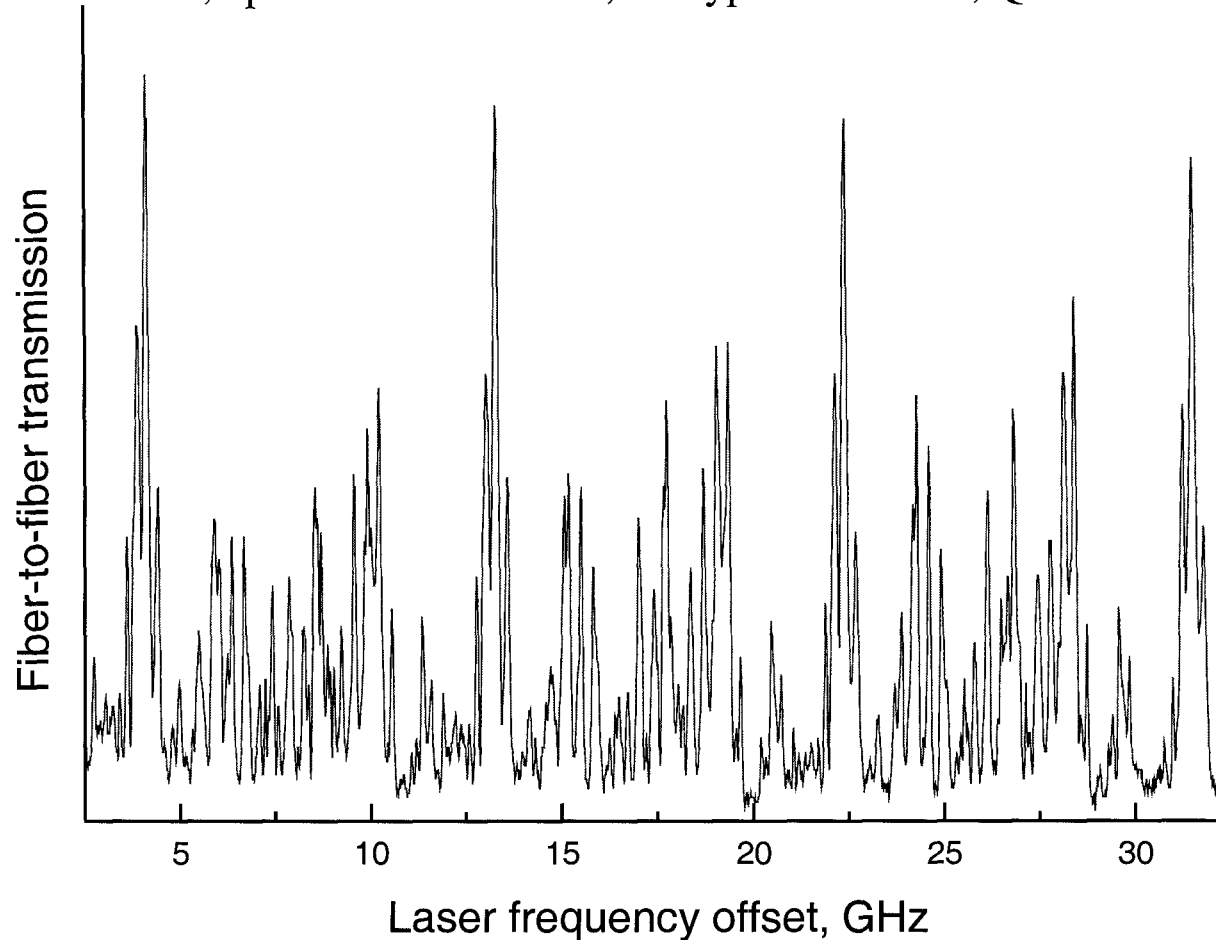
The setup is completed with microwave coupler(s),  
one or two optical coupling prisms, and fiber collimators



## Typical optical spectrum obtained with single prism and two fiber collimators

separate peak prominence highly dependent a) on launch angle vs crystal axis b) position of pick-up fiber.

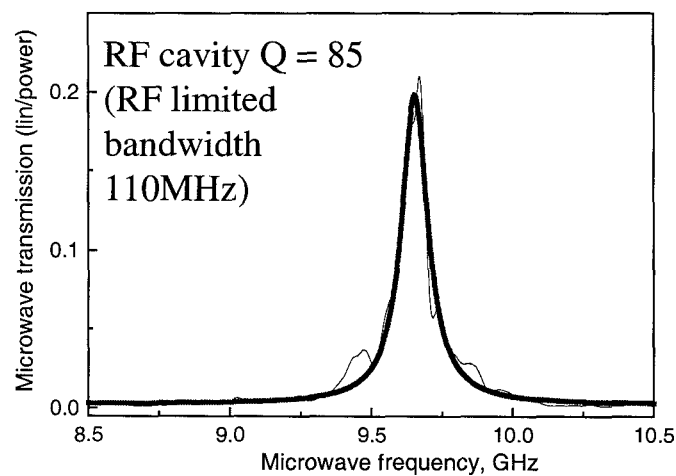
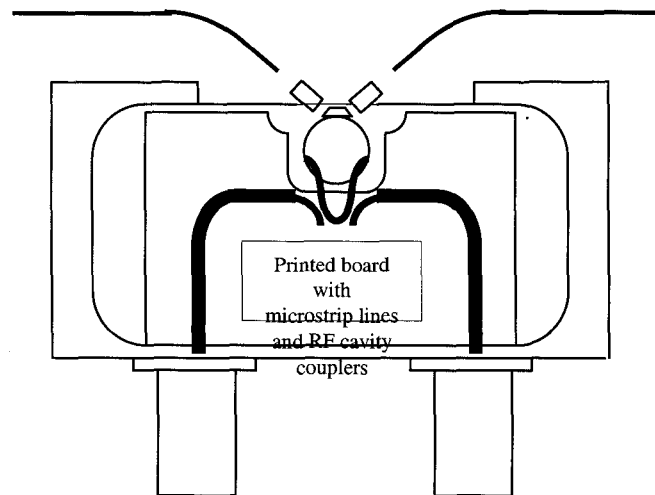
“Disk” dia. 4.8mm, optical FSR  $\sim 9.1\text{GHz}$ , TE-type WG modes,  $Q \sim 3 \times 10^6$  @ 1550nm



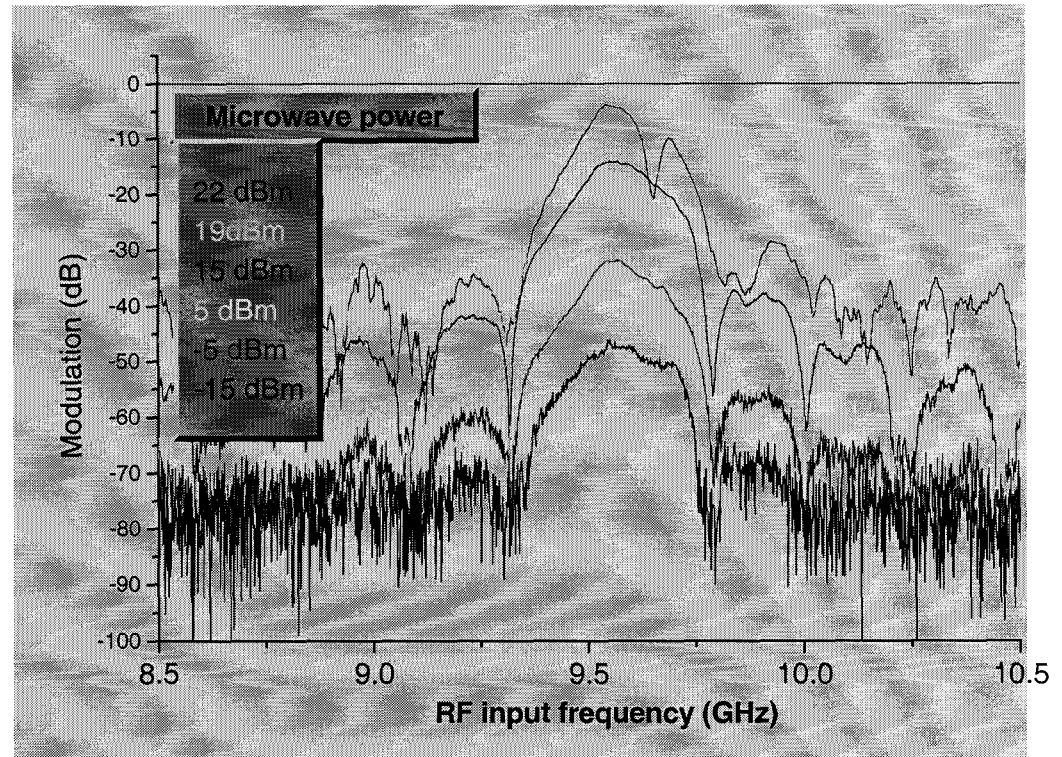


Variant 1: single “horseshoe” MW cavity, “disk” with 4.6mm dia., 0.75mm height

Curved adaptation of a half-wave open-end microstrip-line resonator



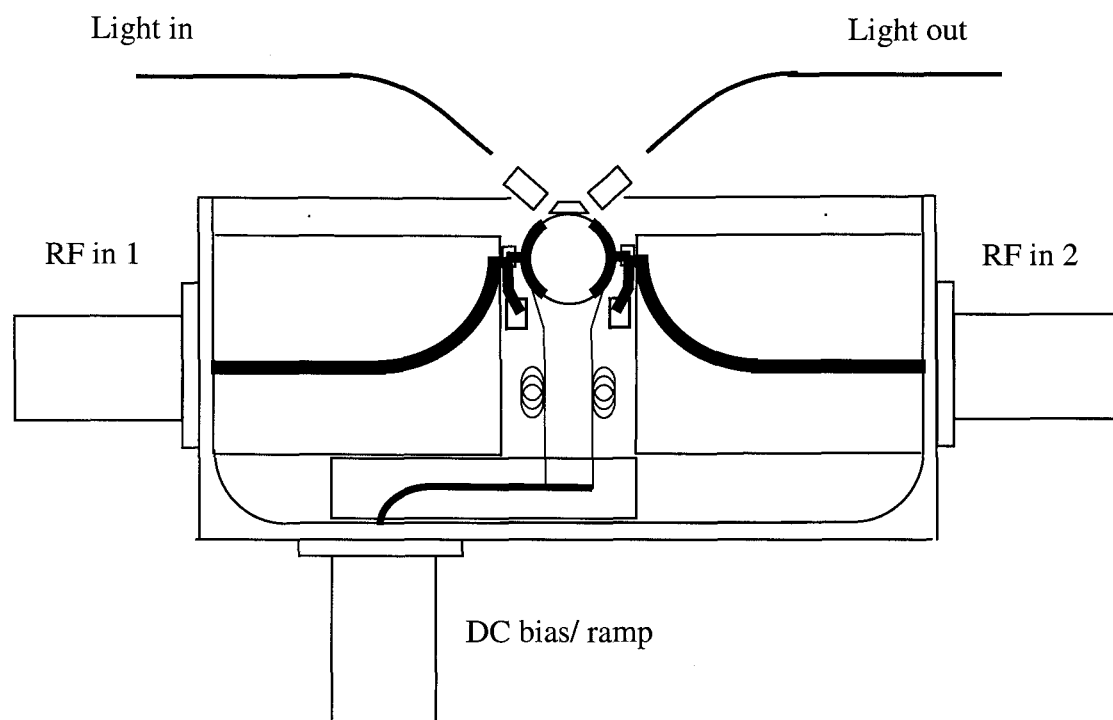
Frequency response of the modulator



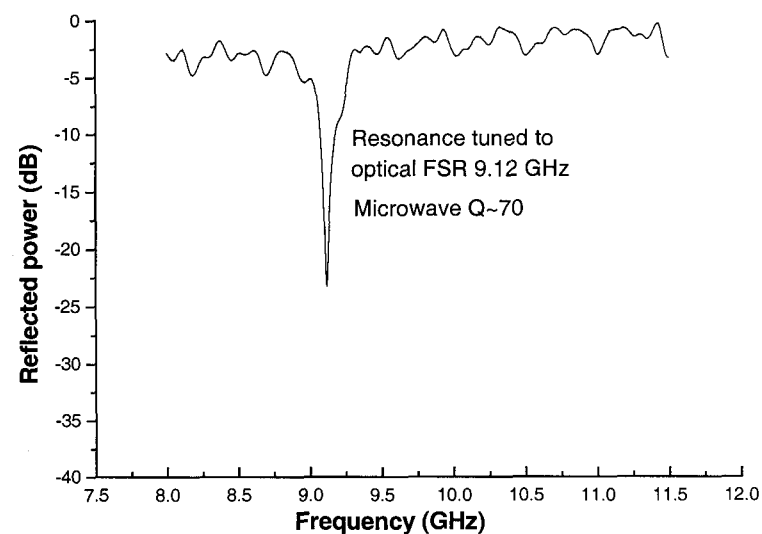


Variant 2 (two tunable MW cavities), “disk” with 4.85mm dia., 0.39mm height

Branched half-wave microstrip cavities tunable by quasi-lumped capacitor at idle end



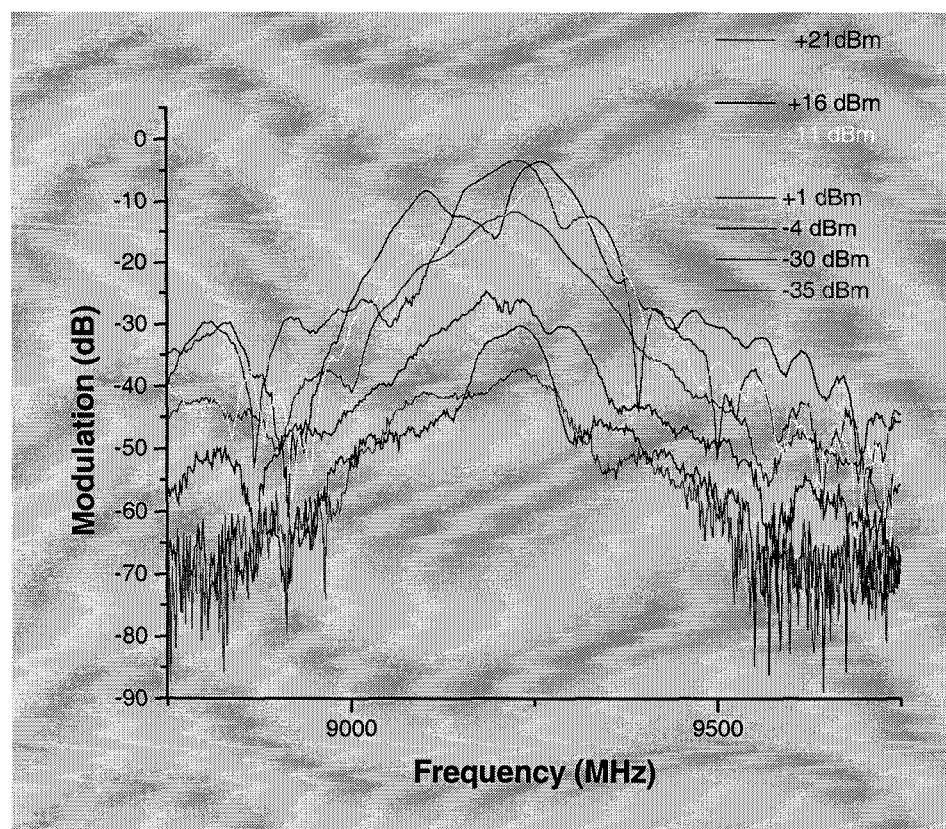
Microwave resonance at port 1



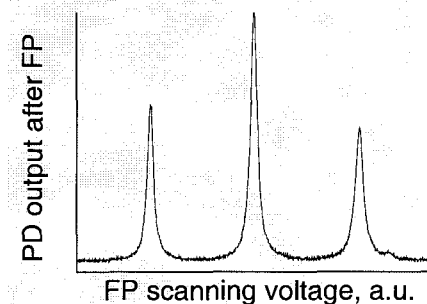


## Variant 2, microwave modulation

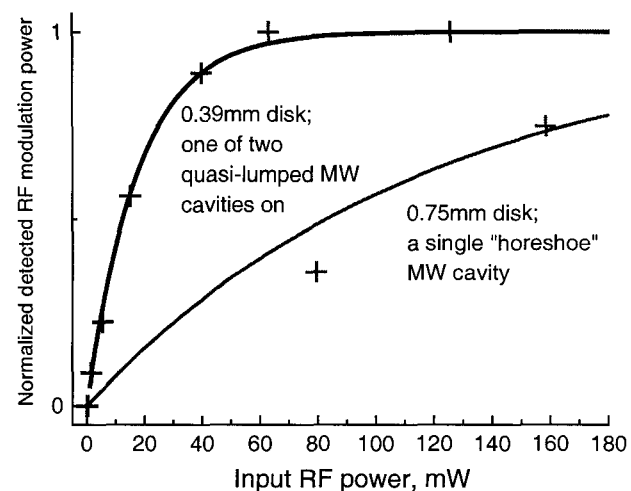
Frequency response of the modulator (one port driven so far)



Optical Spectrum of Modulated Signal  
9.2GHz sidebands maximized by adjusting output coupler



RF power to drive the modulator:  
~40mW with just one RF cavity  
Compare to ~1W in Mach-Zender  
*Lots of room to go*





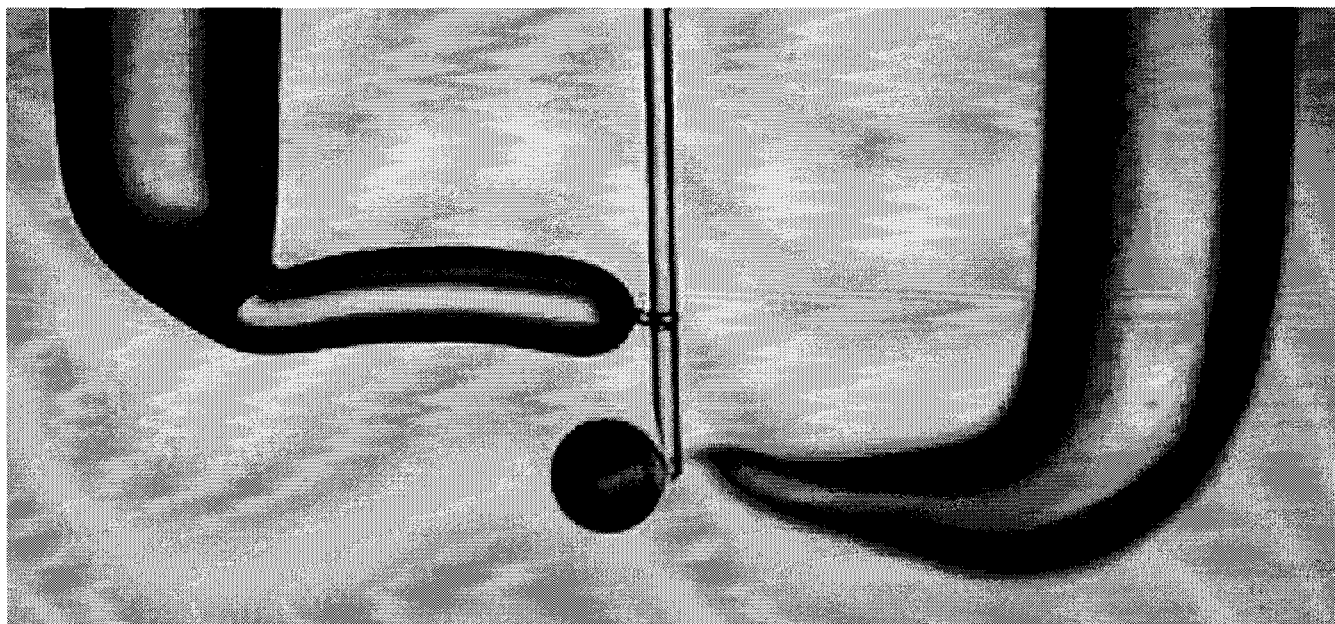


## Electro-optic modulation with whispering-gallery modes (conclusion)

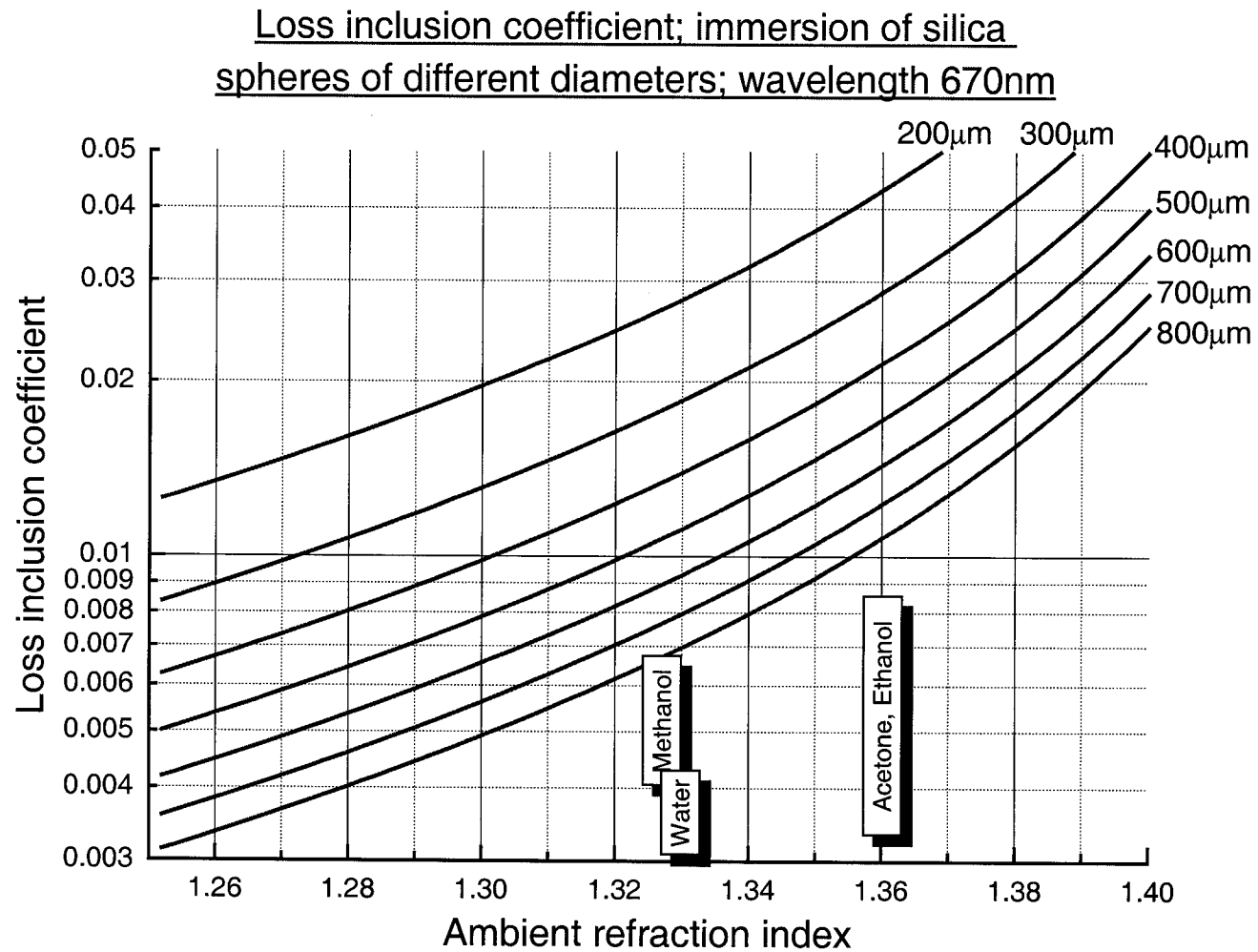
- Optical losses in lithium niobate allow for high-Q  $(0.3-1) \times 10^7$  WG modes. High finesse translates into potential of strong reduction of controlling electrical power compared to zero-order interferometers such as MZ
- RF driving power can be further reduced by matching microwave cavity that can be as high-Q as  $10^3$  (limited by dielectric loss)
- Limited band, non-power-hungry modulators (1mW and less is feasible) may be useful for mm-wave applications such as low-rate data, fiber radio & picocellular com, as well as in novel optoelectronic oscillators



## WG microcavity as a sensor: detection of nanomole dopants in fluids



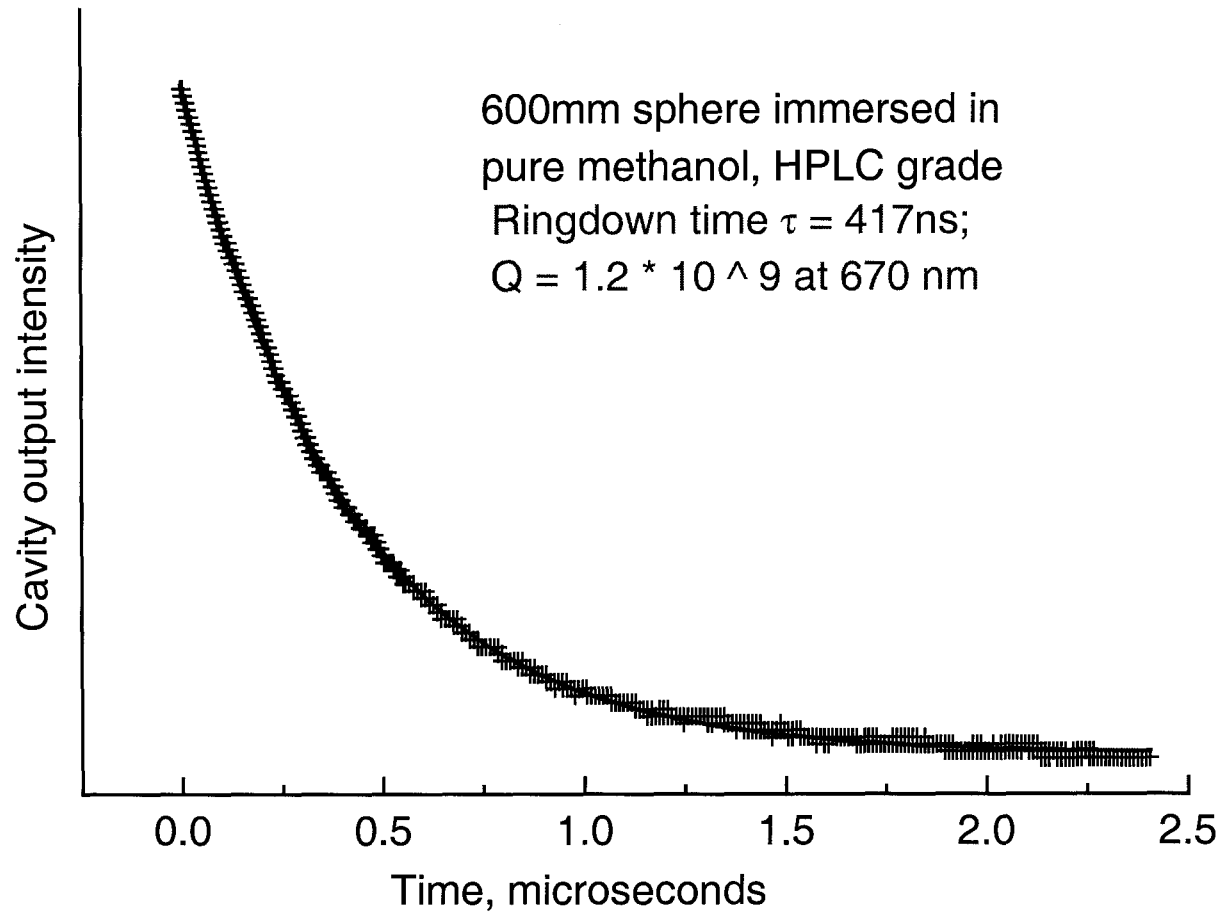
Evanescent wave makes the Q-factor of WG modes a function of optical loss in the ambient. Alternatively, the resonator becomes a miniature replacement of intracavity absorption cell



(Alyrzayev, Gorodetsky, Ilchenko, 1994, Unpublished)

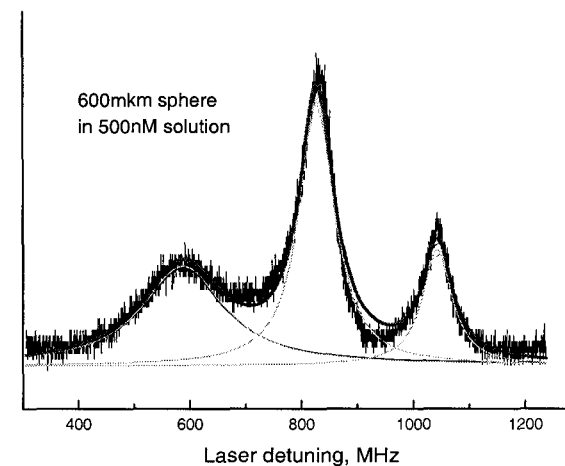
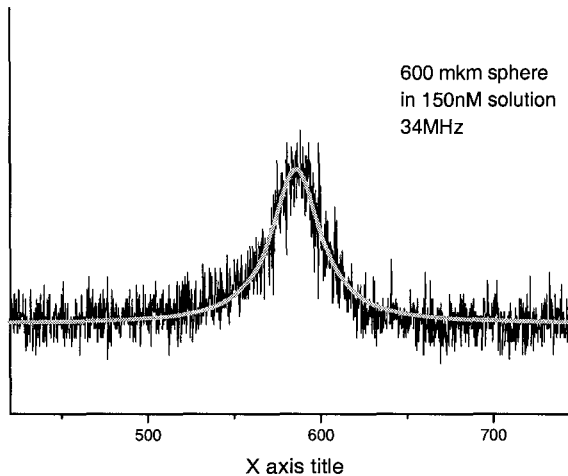
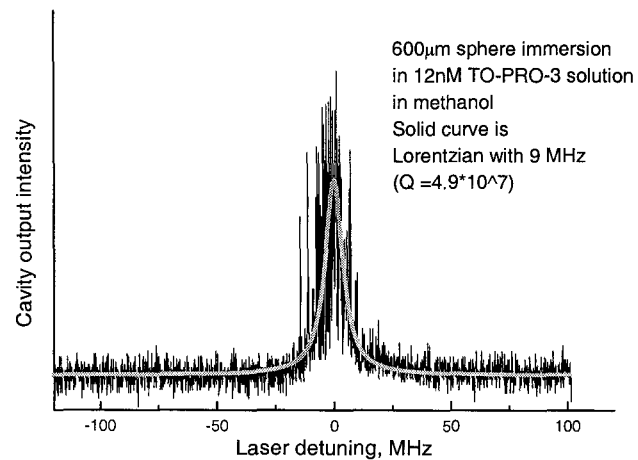
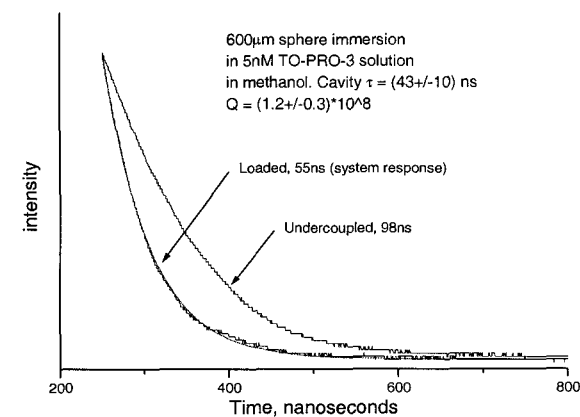
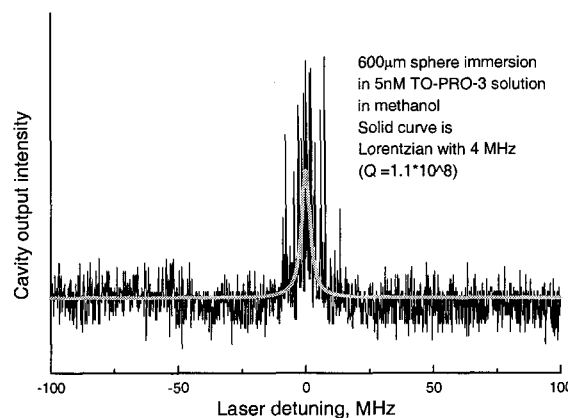
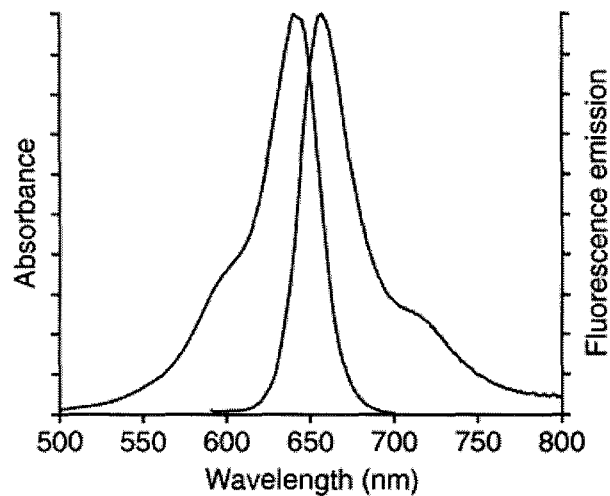


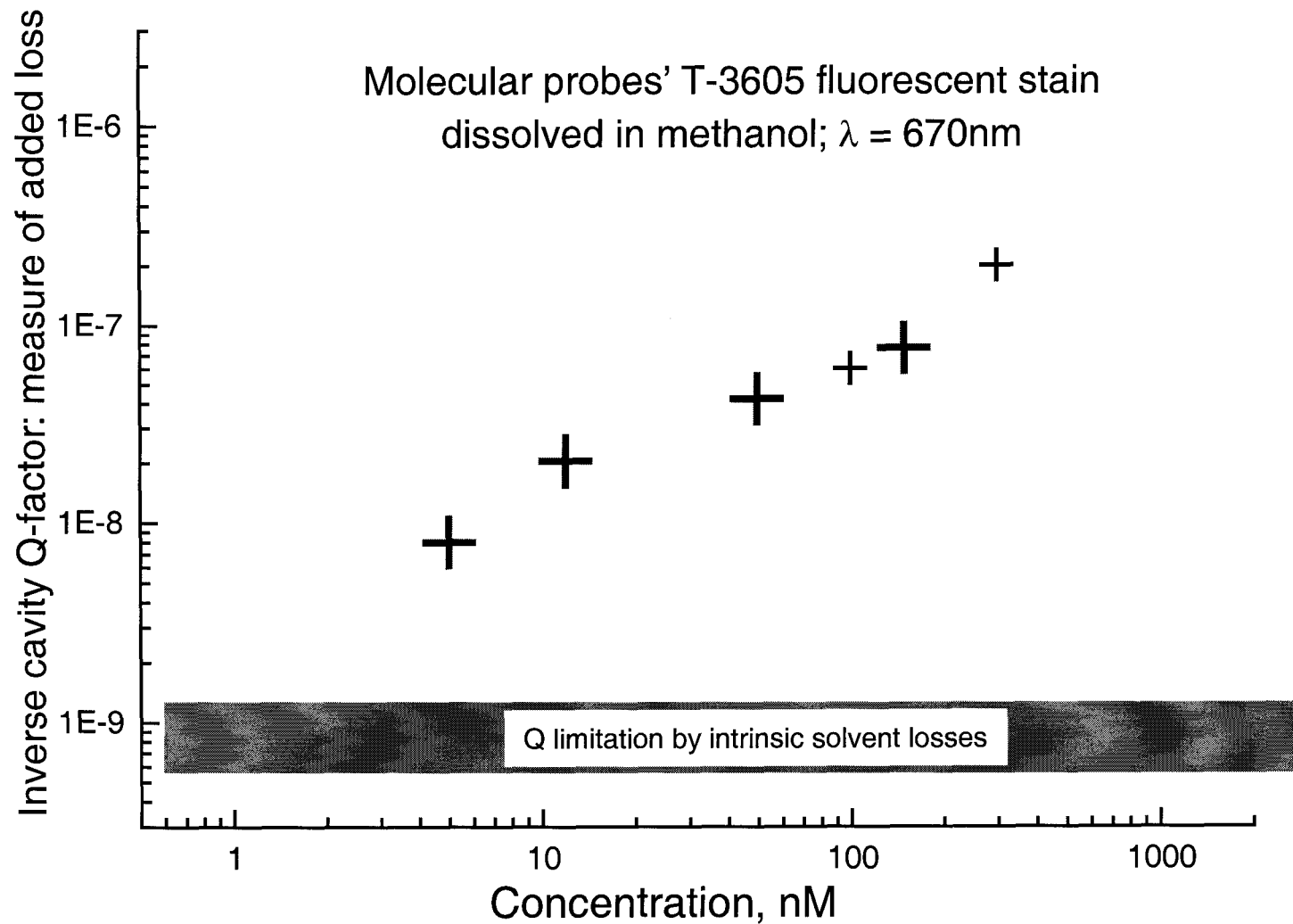
Immersion in pure solvent: background loss allows  $Q \sim 10^9$





## Demonstration: dissolving TO-PRO-3 fluorescent dye







## CONCLUSION: WG microcavity as a sensor

- Immersed microsphere cavity is a novel micro multi-path cell for detection and, later, (cavity ringdown) spectroscopy of very dilute species
- Detectable concentrations of analytes, currently at nanoMole levels, can be reduced to sub-picoMole -- in a pure added-absorption method. Actual sampled volume is in the range of  $10^{-9} \text{ cm}^3$  (“attoliter”):  
-- just thousands of molecules act
- Choice of solvents may not be limited to  $n < n_{\text{silica}}$  :  
immersed sphere can be inverted



## CONCLUSION (General)

High-Q microcavities with whispering-gallery modes  
reveal continued capabilities for research and applications